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Reconstruction of Probabilistic Distributions of Multivariate Random Functions from Distributions of Their Projections

V.G. Ushakov and O.V. Shestakov

Moscow State University
e-mail: ushakov@comtv.ru
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The imaging techniques of reconstruction from projections have found wide applications in many fields of science. The theoretical foundation for these techniques was developed by Radon in 1917, who proved that function, describing inner structure of an object, can be uniquely reconstructed from all of its projections. And he actually proposed a reconstruction formula, which can be considered as an algorithm of reconstruction. Sometimes, however, this conventional tomographic approach cannot be used directly. An object can have several states (even an infinite set of states) which randomly change during the process of projection acquisition. The problems of this kind resulted in appearance of a new field of tomography - stochastic tomography. In stochastic tomography the problem of reconstruction from projections is studied in the case when an object under investigation is described by a random function, and different projections correspond to its different realizations. We consider the problem of determination of probabilistic characteristics of such objects if characteristics of projections are known.

As for mathematical expectation (or average of distribution) the problem appears to be equivalent to conventional (nonstochastic) tomography. But situation changes for problem of determination of other probabilistic characteristics. It appears that in a general case this problem is characterized by great ambiguity. For instance (and there are numerous examples of that), it appears that, in a general case, knowing variances of all projections is not enough to reconstruct variance of a random function. Moreover, variances of projections do not tell anything even on a qualitative level and even in simplest cases (in fact, even if random function takes only two values in each point). That is a point with largest variance within an object may have projections with the least variance and visa versa. It also appears that in a general case one cannot reconstruct variance even if all joint distributions of each projection are known. However, if some restrictions on random functions, describing an object, are introduced, one can reconstruct probabilistic characteristics of random function from distributions of its projections. We considered a case when an object is described by a random function that has at most denumerable number of realizations (states) and each realization has a support on a compact set, which adequately reflects the reality since all objects in real life have finite dimensions and can be only in a finite number of states. In this case it was proved that there is a unique correspondence between probabilistic distribution of random function, describing an object, and distributions of its projections.

An algorithm of reconstruction of probabilistic distributions of multivariate random functions from distributions of their projections was developed. This algorithm is based on the properties of so-called moments of projections, which can be represented by finite Fourier series of angle. Taking into account the form of the moments, one can sort registered projections by the groups corresponding to different realizations of the random function. When all projections are sorted, one can reconstruct each realization of the random function (and hence its distribution), using conventional techniques of computer tomography. We also studied influence of inaccuracies in projection data on algorithm of reconstruction and obtained some estimates of errors in reconstructed characteristics of a random function.